ACCURATE SOLUTION DESCRIBING AXISYMMETRIC GAS-DYNAMIC STRUCTURES

M. A. Demidov and A. P. Mikhailov UDC 533.6.011

An accurate solution describing the dynamics of complex gas-dynamic structures arising in the adiabatic compression of material in conditions with aggravation is constructed.

Impactless supercompression of a finite mass of gas [1-22] occurs if the pressure at the boundary of the material (piston) increases in the following conditions with aggravation

$$p(x_p,\ t) = p_0(t_f - t)^{n_S} \to \infty,\ t \to t_f,\ n_S = -2\gamma(N+1)/(\gamma+1+N(\gamma-1)), \tag{1}$$

where $t_f < \infty$ is the instant of aggravation; N = 0, 1, 2.

Boundary conditions with aggravation in gas-dynamics problems were studied in [13-20], and effects of localization and formation of gas-dynamic structures (temperature or density extrema associated with fixed gas particles) in media with an arbitrary entropy distribution for the cases N = 0, 2 in [17-19].

It is shown in the present work that, for all axisymmetric (N = 1) self-similar solutions in separable variables (which, in contrast to solutions in characteristics [1, 7-10], admit of generalization to a broad class of physical processes [2, 3, 6, 11, 12]), the pressure is a linear function of the mass coordinate. In the case of adiabatic compression of ideal gas by a piston in Eq. (1), this allows an accurate solution describing, in particular, the dynamics of complex structures in a compression wave to be found. Numerical calculations demonstrate the stability of the solutions obtained, the achievement of self-similar conditions of compression, and methods of exciting complex gas-dynamic structures.

Separation of the time and mass variables [2, 3, 6, 11, 12] in the equations of axisymmetric gas motion

$$\frac{\partial^2 r}{\partial t^2} = -r\frac{\partial p}{\partial x} \tag{2}$$

leads to two ordinary differential equations

$$p_2'(x) = \lambda,\ r_1'' r_1^{-1} p_1^{-1} = -\lambda,\ p = p_1(t)p_2(x),\ r = r_1(t)r_2(x) \tag{3}$$

where λ is the separation constant.

It follows from Eq. (3) that

$$p(x,\ t) = p_1(t)(\lambda x + C),\ C = \mathrm{const}. \tag{4}$$

Hence, apart from the dependence on the properties of the medium and also on the equations of energy balance and continuity of all the flows described by self-similar solutions in separable variables, a linear dependence of the pressure on the mass coordinate is characteristic. If a potential force also appears in the equation of motion in Eq. (2), an analogous conclusion is valid for the quantity $\Sigma = p + \Pi$ (Π is the potential).

In the case of adiabatic flow of ideal gas, it is necessary to add to Eq. (2) the adiabaticity integral and the continuity equation

$$\frac{\partial\eta}{\partial t} = \frac{\partial(rv)}{\partial x},\ \frac{\partial v}{\partial t} = -r\frac{\partial p}{\partial x},\ \frac{\partial r}{\partial t} = v,\ p\eta^\gamma = \varphi^\gamma(x), \tag{5}$$

where $\varphi(x)$ is an arbitrary summable function describing the entropy distribution over the mass of the material.

M. V. Keldysh Institute of Applied Mathematics, Academy of Sciences of the USSR, Moscow.
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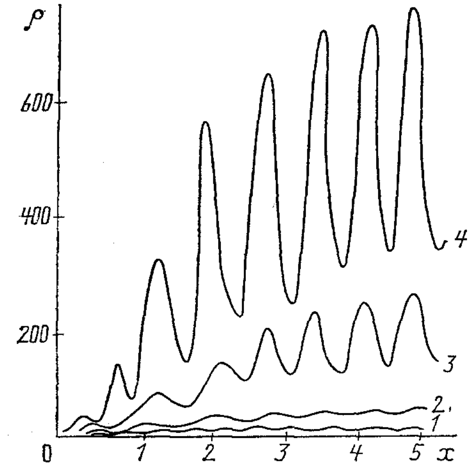

Fig. 1

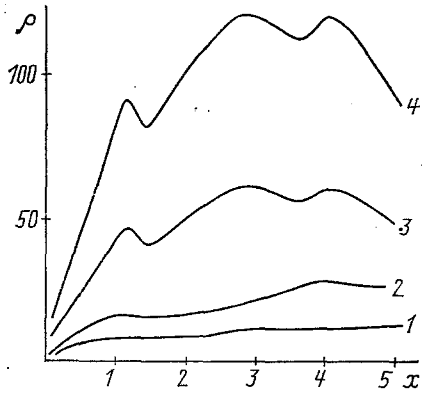

Fig. 2

Fig. 1. Dynamics of complex gas-dynamic structures ($p^{\circ} = 1$, $v^{\circ} = 0$, $\alpha = 8\pi/x_p$, $x = 5.0$, $\gamma = 5/3$): $t_1 = 0.722$; $t_2 = 6.930$; $t_3 = 0.979$; $t_4 = 0.992$ (curves 1-4, respectively).

Fig. 2. Excitation of complex gas-dynamic structures ($p^{\circ} = \rho^{\circ} = 1$, $v^{\circ} = 0$, $x_p = 5.0$, $\gamma = 5/3$): $t_1 = 0.711$; $t_2 = 0.863$; $t_3 = 0.917$; $t_4 = 0.942$ (curves 1-4, respectively).

Consider the problem of gas compression by a piston at the point $x = x_p$, the pressure at which increases in the conditions with aggravation in Eq. (1), i.e., one of the solutions of the system in Eq. (3).

The property in Eq. (4) allows an accurate solution of Eqs. (1) and (5) to be obtained:

$$\begin{gathered} p(x\quad t) = p_0 (t_f - t)^{-2} (\lambda x + C),\ \lambda = (1 - C)/x_p, \\ v(x,\ t) = v_0 (t_f - t)^{-(\gamma-1)/\gamma} v_2(x),\ v_2(x) = \mp \sqrt{\int 2\varphi(x)(\lambda x + C)^{-1/\gamma} dx}, \\ r(x,\ t) = r_0 (t_f - t)^{1/\gamma} r_2(x),\ r_2(x) = \pm v_2(x),\ \eta(x,\ t) = \varphi(x) p^{-1/\gamma}(x,\ t), \end{gathered} \tag{6}$$

where the constant of integration in the expression for the velocity is chosen from the condition $v(0) = r(0) = 0$. The parameter λ is calculated from Eq. (1); the quantity $0 \leq C < 1$ determines the pressure at the symmetry center of the system $p(0, t) = Cp(x_p, t)$. If $C < 0$, however, the pressure vanishes at some point $x^* = -C/\lambda < x_p$ and the solution in Eq. (6) describes gas compression with an internal cavity collapsing as $t \to t_f$ to the symmetry axis of the system.

The inhomogeneous entropy distribution in the medium and the development of conditions with aggravation, as shown in [13, 15-19], lead to the appearance of structures in the compressible material. In the given flows, the condition of existence of gas-dynamic density ($\rho = \eta^{-1}$) and temperature ($T = p\eta R^{-1}$) structures is found from Eq. (6) in an explicit form

$$\frac{d\varphi}{dx} = \frac{\gamma\lambda}{\lambda x + C}\ (\text{for}\quad \rho),\quad \frac{d\varphi}{dx} = -\frac{\gamma}{\gamma - 1}\frac{\lambda}{\lambda x + C}\quad (\text{for}\quad T). \tag{7}$$

If $\varphi'(x) \equiv 0$, there are no structures (isoentropic compression [2, 14]); if $\varphi'(x) > 0$, only density structures may exist and if $\varphi'(x) < 0$ only temperature structures. The number of extrema is completely determined by the slope of $\varphi(x)$; hence, even with a monotropic entropy distribution in a compressible medium, all possible configurations of temperature or density extrema may exist.

Thus, the solution in Eq. (6) describes gas flow with the following interesting properties (S conditions [2, 3, 6, 11-20]): 1) the material is compressed in conditions with aggravation: as $t \to t_f$, the pressure, density, and valocity of the medium increase indefinitely for all $0 \leq x \leq x_p$; 2) compression occurs without the appearance of shock waves; 3) complex gas-dynamic structures exist in the compression wave.

The spatial characteristics of Eq. (6) are valid for all adiabatic flows with a linear velocity profile over the radius [21, 22], since Eq. (4) holds for all axisymmetric flows with homogeneous deformation.

Numerical calculations of Eqs. (1) and (5) by the method of [23] show that the self-similar solutions in Eq. (6) are stably reproduced with increase in pressure at the piston by a factor of 10^7. For the achievement of S conditions of compression (the establishment of a linear pressure profile in the mass coordinates and stabilization of the wave halfwidth), approximately 25-fold increase in pressure at the piston boundary is sufficient; see [20], for example.

The dynamics of complex gas-dynamic structures is shown in Fig. 1. The initial pressure and velocity distributions are homogeneous; the density is specified in the form $\rho(x, 0) = 2 + \cos(\alpha x)$. The shock wave arising because of non-self-similarity of the initial data accumulates at the symmetry axis and is then reflected. Consequently, there is a change (although weak) in the coordinates and amplitudes of the initial density extrema (times t_1, t_2). Then under the action of boundary conditions with aggravation

$$p(x_p, t) = (1 - t)^{-2} \tag{8}$$

stable density structures described by Eq. (6) are formed (t_3, t_4).

One method of exciting complex gas-dynamic structures with homogeneous initial data and monotonic boundary conditions in Eq. (8) is shown in Fig. 2. On account of the initial mismatch of the pressure at the piston and in the medium — $p(x_p, 0)/p(x, 0) = 50$ — a shock wave is formed and then accumulates and is reflected from the symmetry axis of the system (t_1, t_2); this leads to an inhomogeneous entropy distribution. Further increase in pressure at the piston in conditions with aggravation in Eq. (8) ensures adiabatic supercompression of the material (without the appearance of new shock waves) and the formation of stable gas-dynamic structures of S conditions (t_3, t_4); see the solution in Eq. (6).

NOTATION

p, v, η, T, ρ, pressure, velocity, specific volume, temperature, and density of the material; t, time; x, mass coordinate; r, particle radius; γ, adiabatic modulus; N, symmetry index; R, gas constant.

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SUPERSONIC FLOW OVER A SPHERE IN THE WAKE OF A BLUNT CYLINDER

Yu. P. Golovachev and N. V. Leont'eva

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The authors use numerical solutions of the Navier–Stokes equations to investigate the influence of geometric factors on the flow structure ahead of a sphere and the surface distributions of pressure and heat transfer.

References [1-6] have conducted experimental and theoretical investigations of supersonic flow over bodies located in a wake region. It has been established that a nonuniform distribution of the incident flow parameters has an appreciable influence on the bow shock shape, the boundary layer structure, and the distribution of pressure, friction stress and heat transfer. It has been indicated that reverse circulation flows may form on the forward surface of blunt bodies. In the theoretical studies the wake flow was modeled by different shear flows. In the present paper the parameters of the flow incident on a sphere are adopted from the solution of the problem of the supersonic wake behind a blunt cylinder obtained in [7], using the Navier–Stokes equations under the flow symmetry hypothesis. We investigate how the shock layer structure and the distribution of drag and heat transfer on the surface of the sphere depend on the distance between the bodies and their radii.

1. We consider stationary axisymmetric flow over a sphere whose center is located on the axis of the supersonic wake behind a longitudinally washed blunted cylinder. The computation region is bounded by the body surface, the axis of symmetry, the bow shock, and a certain surface on which the normal velocity component of the gas is everywhere supersonic apart from a narrow wall region. The flow is described by the full Navier–Stokes equations. The specific heat of the gas is considered constant. The temperature dependence of the viscosity is approximated by the function $\mu \sim \sqrt{T}$, and the Prandtl number Pr = 0.7.

As boundary conditions on the bow shock we use the generalized Rankine–Hugoniot relations. On the body surface we assign conditions of no slip, impermeability, and either a constant temperature or a thermal insulation condition. The other two boundaries have, respectively, the symmetry conditions and approximate boundary conditions based on the hypothesis of a sufficiently smooth solution with respect to the angular coordinate. The stationary solutions are found by a time-dependent method. As initial data in most cases we assume the results of solving the problem for the close variant of the flow conditions.

2. The full unsteady Navier–Stokes equations for a compressible gas in spherical coordinates have been given in [7]. In solving the problem the distance from the sphere surface is normalized by the shock standoff distance. As a result the shock layer region examined is transformed into a rectangle. The original equations are written for the vector of the desired functions $X = \{u\ T\ v\ p\}^T$ in the form

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